TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 444

WORKING CHARTS FOR THE STRESS ANALYSIS OF ELLIPTIC RINGS

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SUMMARY

This report presents charts which reduce the stress
analysis of circular and elliptic rings of uniform cross
section subjected to balanced systems of concentrated
loads from a statically indeterminate problem to a stati-
cally determinate one. The charts are constructed for two
simple loading conditions into which most of the loading
conditions encountered in the design of the main frames of
a monocoque fuselage may be resolved.

The theoretical calculations required for the con-
struction of the charts employed the "minimum energy meth-
od" but are omitted in the report.

To demonstrate the use of the charts in the stress
analysis of elliptic rings an illustrative problem is in-
cluded.

INTRODUCTION

The design of main frames for monocoque fuselages has
been discussed by Roy A. Miller in reference 1, where he
also developed equations for the shear, thrust, and moment
in a circular ring of uniform cross section subjected to
balanced systems of concentrated loads symmetrical with
respect to a diameter. As the monocoque fuselage is per-
haps more often elliptical than circular in section, the
author has extended the solutions given in reference 1 to
include the elliptic ring of uniform cross section subject-
ed to balanced systems of concentrated loads symmetrical
with respect to the major axis.

From structural theory and dimensional reasoning it
developed that for any loading condition the shear, thrust,
and moment at any point in a ring could be reduced to co-
efficient form and presented conveniently in charts, and
that with a series of these charts the stress analysis of

the ring is reduced from a statically indeterminate problem to a statically determinate one. The object of this report is to present a group of these charts for circular and elliptic rings which will be useful in the stress analysis of the main frames in monocoque fuselages.

The calculations required to evaluate the coefficients plotted in the charts employed the "minimum energy method" and were made during a study of "Elliptic Integrals" under Mr. D. K. Kazarinoff at the University of Michigan, whose valuable assistance and suggestions are gratefully acknowledged. Because the calculations were long and contained nothing new in principle, they have been omitted in the report. However, all the calculations were checked in detail by the National Advisory Committee for Aeronautics, so that the published charts may be considered accurate within the limits of the usual assumptions in structural theory.

To demonstrate the use of the charts in the stress analysis of elliptic rings subjected to balanced systems of concentrated loads symmetrical with respect to the major axis of the ellipse, an illustrative problem is included.

CHARTS

From structural theory and dimensional reasoning, it follows that the shear, thrust, and moment, respectively, at any point in a ring subjected to any system of external loads may be given by the following equations:

$$V = K W \tag{1}$$

$$P = K' W \tag{2}$$

$$M = K'' a W \tag{3}$$

where a is a dimension of the ring, K, K', and K'' are constants depending upon the shape of the ring and the distribution or position of the loads, and W is the load on the ring. For a circular or elliptic ring subjected to concentrated loads W, balanced and symmetrical as shown in Figure 1, K is zero at the ends of the axis of symmetry. The values of K' and K'' at one end of the axis of symmetry are given in Figures 2, 3, 4, and 5 for the two cases shown in Figure 1. These figures are self-ex-

planatory and require no comment except that in these
charts the position of the loads, W, is given by Φ,
values of which are plotted against θ for the various
a/b ratios in Figure 6.

LIMITATION OF THE CHARTS

The charts are derived for thin rings, rings in which
the dimensions of the cross section are small as compared
to the dimensions of the ellipse, but not so small that
deformation under load becomes appreciable and changes in
curvature cannot be closely approximated by the second de-
rivative. Consequently, the charts must be considered as
approximate only. For rings of the proportions usually
encountered in the design of main frames for monocoque fu-
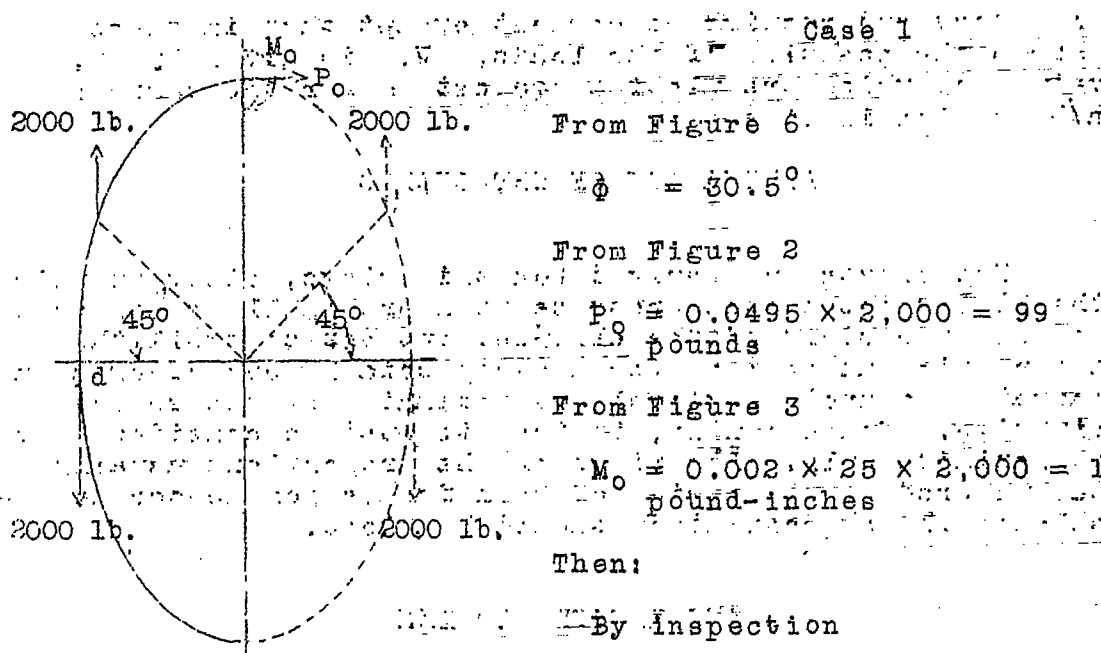selages the approximation should be close.

ILLUSTRATIVE PROBLEM

Given: An elliptic ring
a = 25 inches; b = 14.7
inches; $\frac{a}{b}$ = 1.70 loaded
as shown.
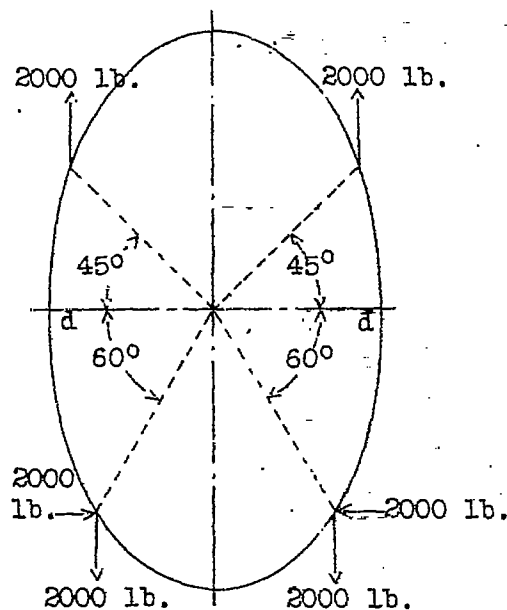
Required: To find the shear,
thrust, and moment (V_d, P_d,
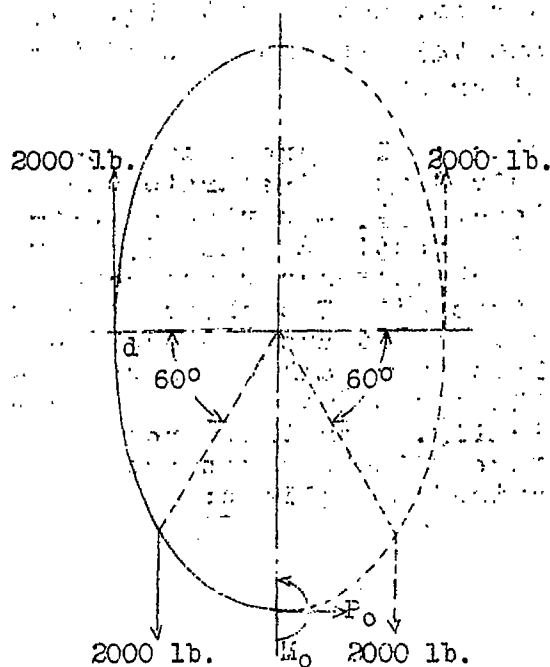and M_d) at the ends of the
minor axis.

Solution: The procedure re-
quired is to separate the
loads into a series of sim-
ple cases, A and B, for
which the charts are con-
structed. The shear, thrust,
and moment are then calcu-
lated for each of these sim-
ple cases and added algebra-
ically. For this problem
three simple cases are re-
quired as follows:

2000 lb.   2000 lb.

45°   45°

d   d

60°   60°

2000 lb.   2000 lb.

2000 lb.   2000 lb.

Case 1

From Figure 6

$$\phi = 30.5°$$

From Figure 2

$P_o = 0.0495 \times 2,000 = 99$ pounds

From Figure 3

$M_o = 0.002 \times 25 \times 2,000 = 100$ pound-inches

Then:

By Inspection

$V_d = 99$ pounds

$P_d = 0$ pound

$M_d = M_o + P_o a - 2,000 (b - b \cos \phi) = 100 + 99 \times 25 - 2,000 (14.7 - 14.7 \times 0.862) = -1,490$ pound-inches

Case 2

From Figure 6

$$\phi = 45.8°$$

From Figure 2

$P_o = 0.096 \times 2,000 = 192$ pounds

From Figure 3

$M_o = 0.029 \times 25 \times 2,000 = 1,450$ pound-inches

Then:

By inspection

$$V_d = -192 \text{ pounds}$$

$$P_d = 2,000 \text{ pounds}$$

$M_d = M_o + P_o a - 2,000 \text{ (b -}$
$\text{b cos } \Phi \text{)} = 1,450 + 192 \times$
$25 - 2,000 \text{ (14.7 - 14.7} \times$
$0.697) = -2,650 \text{ pound-}$
inches

Case 3

From Figure 6

$$\Phi = 45.8^{\circ}$$

From Figure 4

$P_o = -0.92 \times 2,000 = -1,840$
pounds

From Figure 5
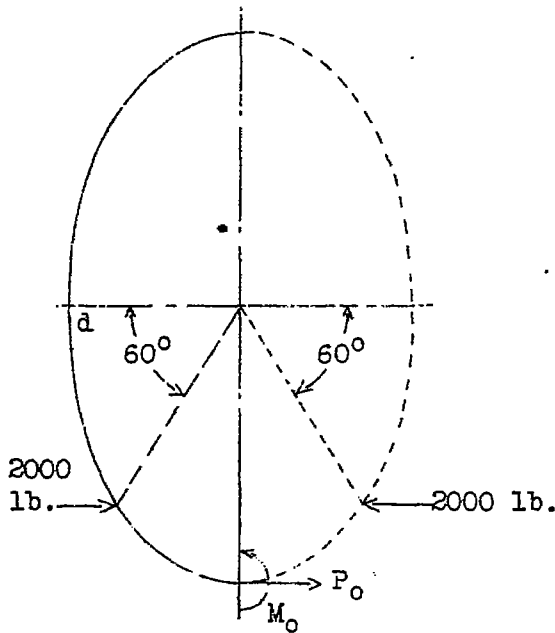
$M_o = 0.168 \times 25 \times 2,000 =$
8,400 pound-inches

Then:

By inspection

$V_d = -(-1,840) - 2,000 =$
-160 pounds

$$P_d = 0 \text{ pound}$$

$M_d = M_o + P_o a + 2,000 \text{ (a sin}$
$\Phi \text{)} = 8,400 - 1,840 \times 25$
$+ 2,000 \text{ (25} \times 0.717) =$
$-1,760 \text{ pound-inches}$

Therefore the shear, thrust, and moment at the ends of the minor axis for the assumed problem are:

	Case 1	Case 2	Case 3	Total
$V_d =$	(99) +	(-192) +	(-160) =	-253 pounds
$P_d =$	(0) +	(2,000) +	(0) =	2,000 pounds
$M_d =$	(-1,490)+	(-2,650) +	(-1,760) =	-5,900 pound-inches

In a similar manner the shear, thrust, and moment can be calculated at any point in the ring. By making a series of such calculations for a number of points, the maximum stress in the ring may be determined.

University of Michigan,

Ann Arbor, Mich., December 8, 1932.

REFERENCE

1. Miller, Roy A.: A Solution of the Circular Ring. Airway Age, May, 1931.
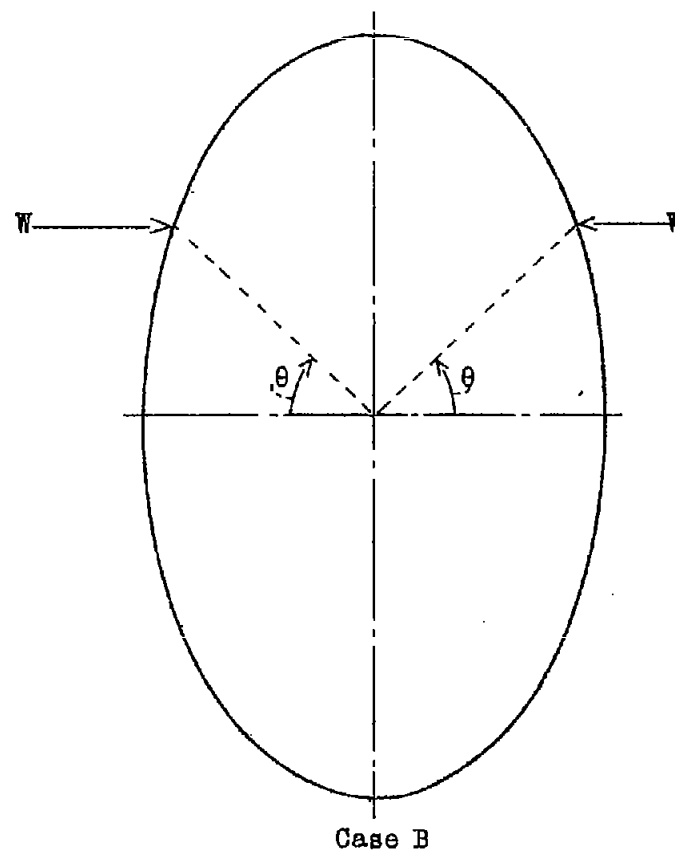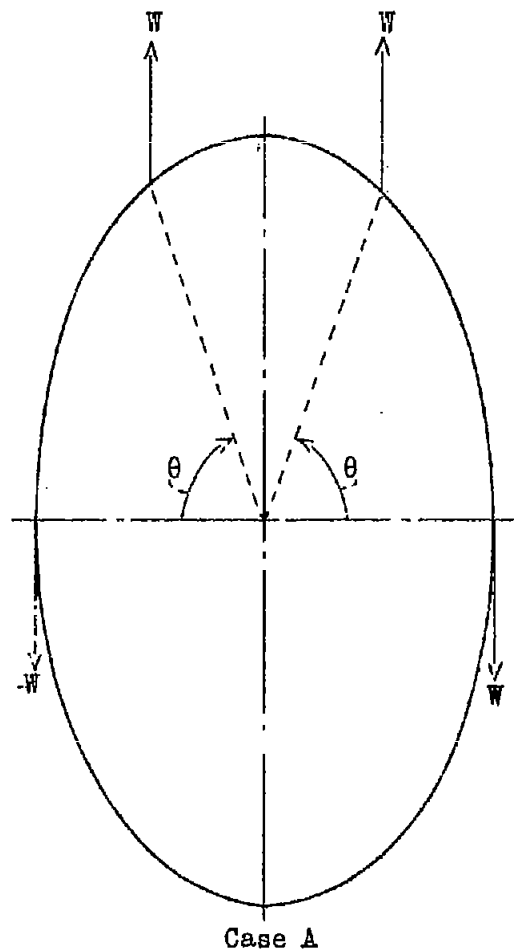
Figure 1.-The two simple loading conditions for which the working charts for the stress analysis of elliptic rings are constructed.
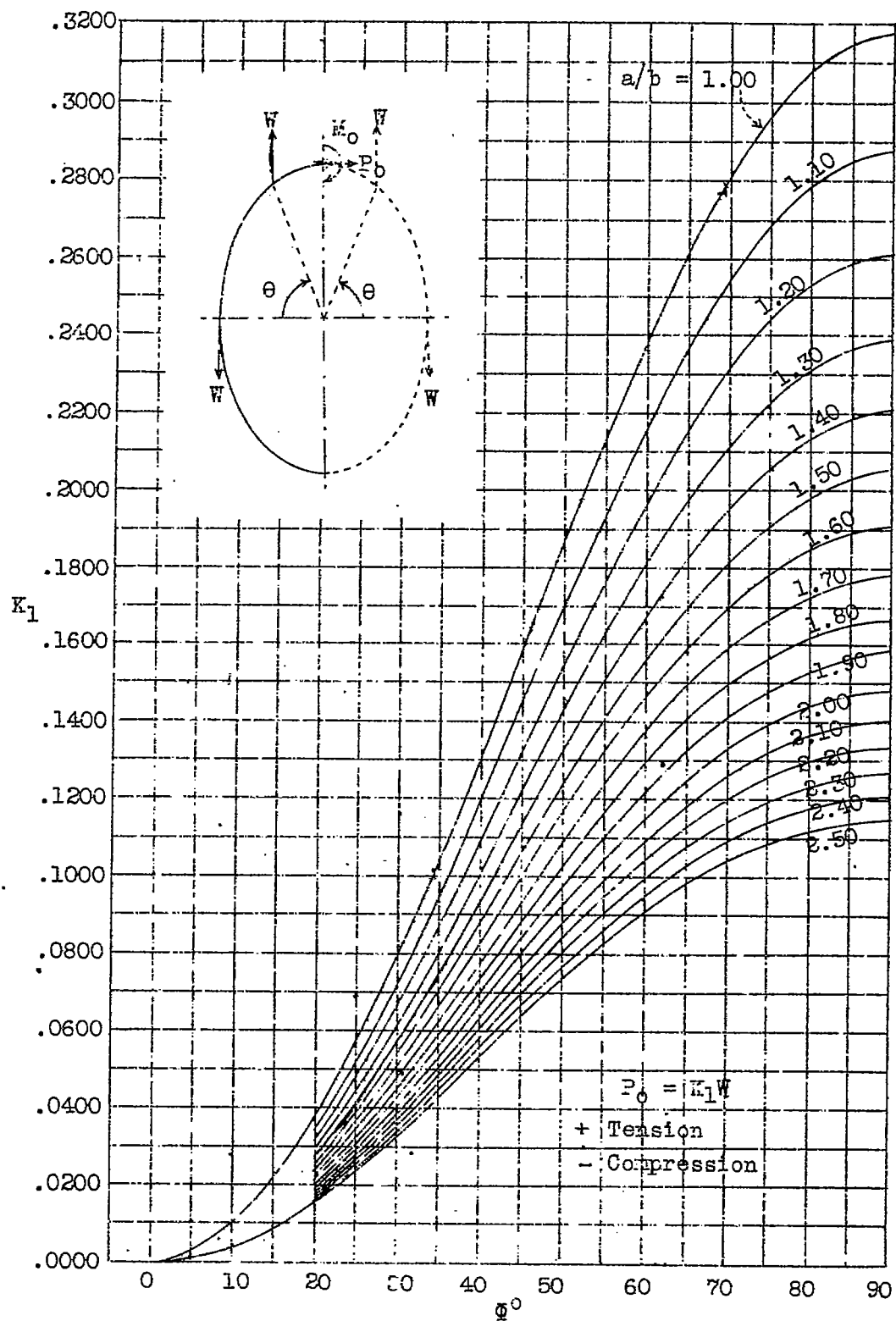
Fig. 1

Fig. 2

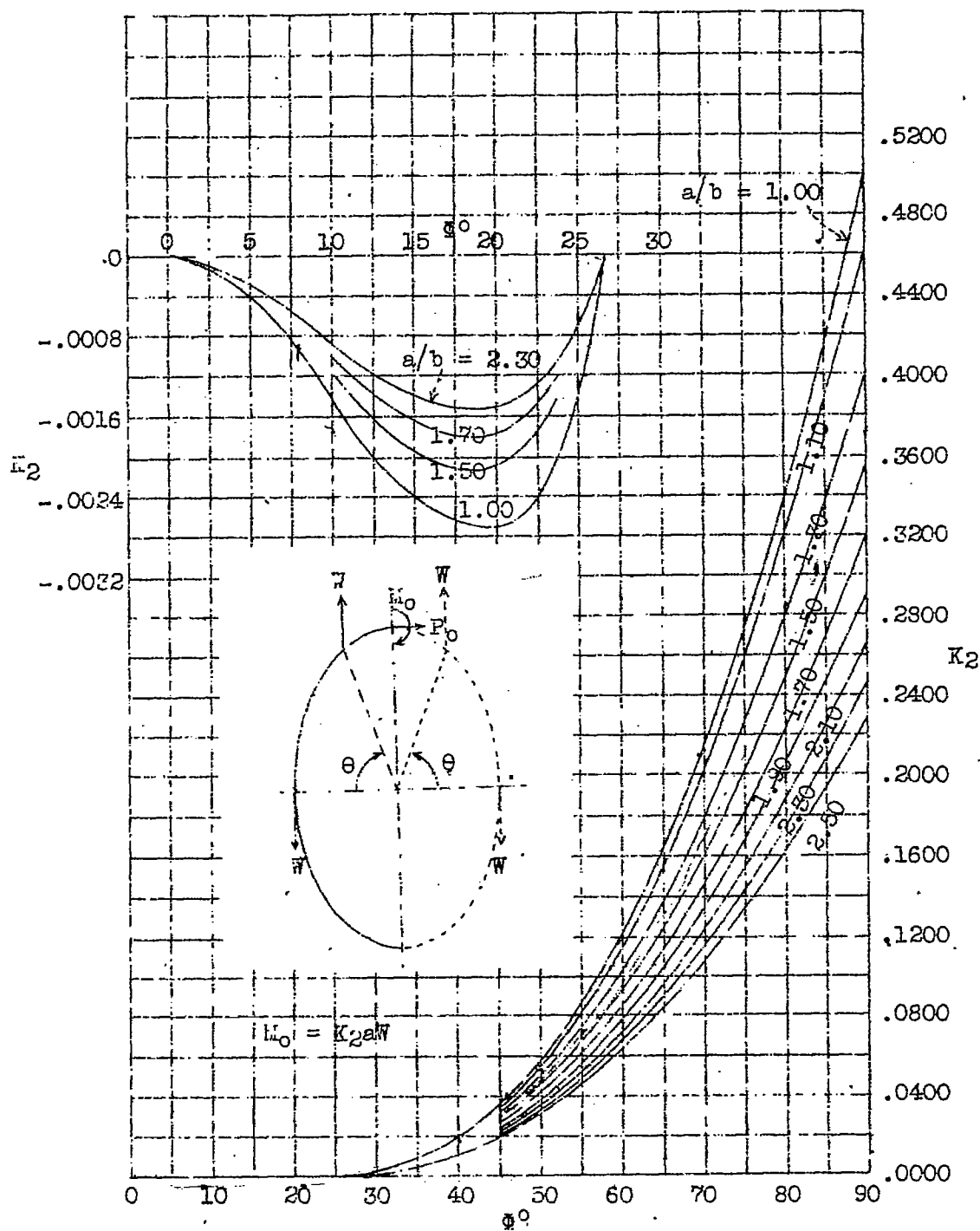Figure 2.- Thrust coefficients for case A.

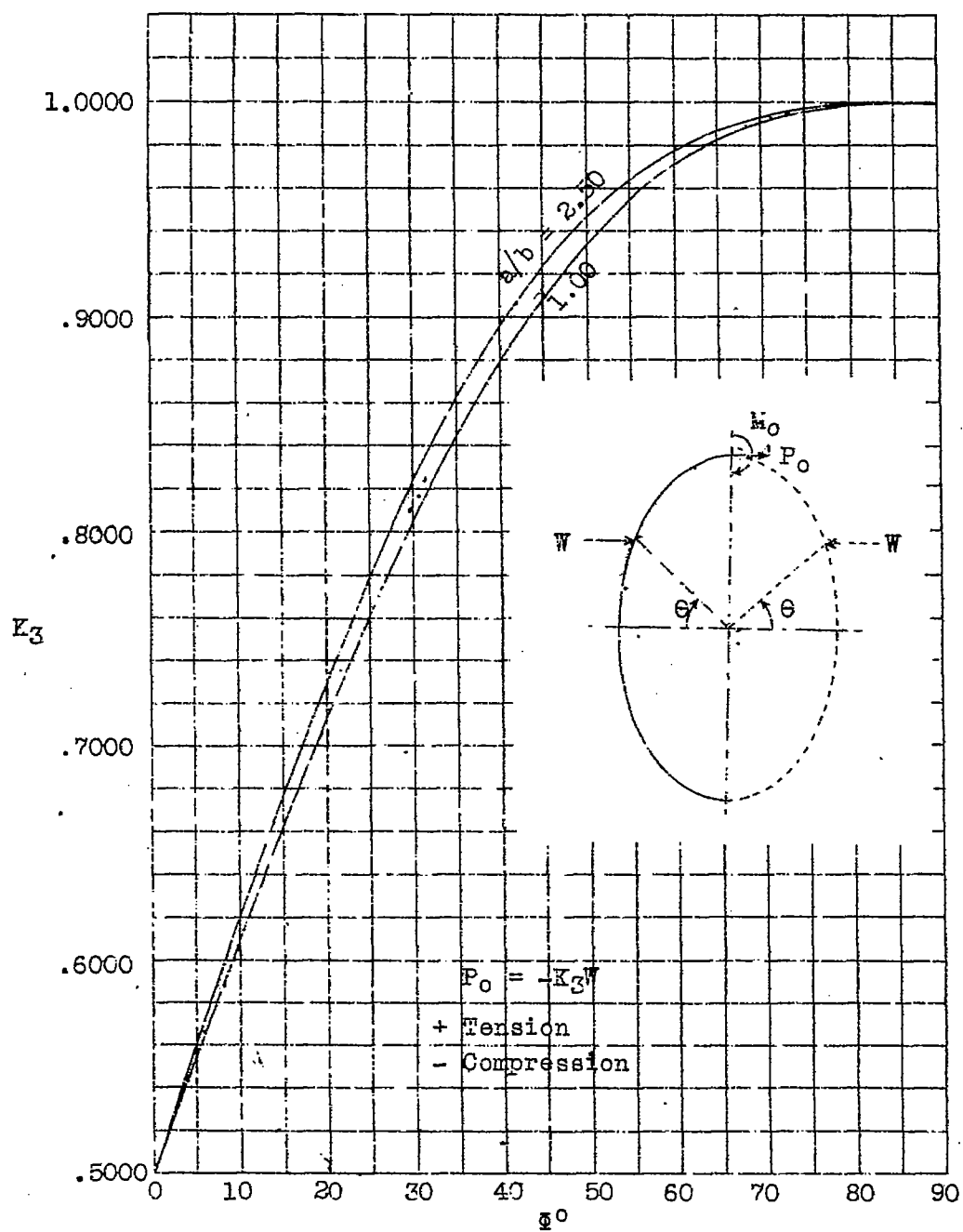Figure 3.- Moment coefficients for case A.
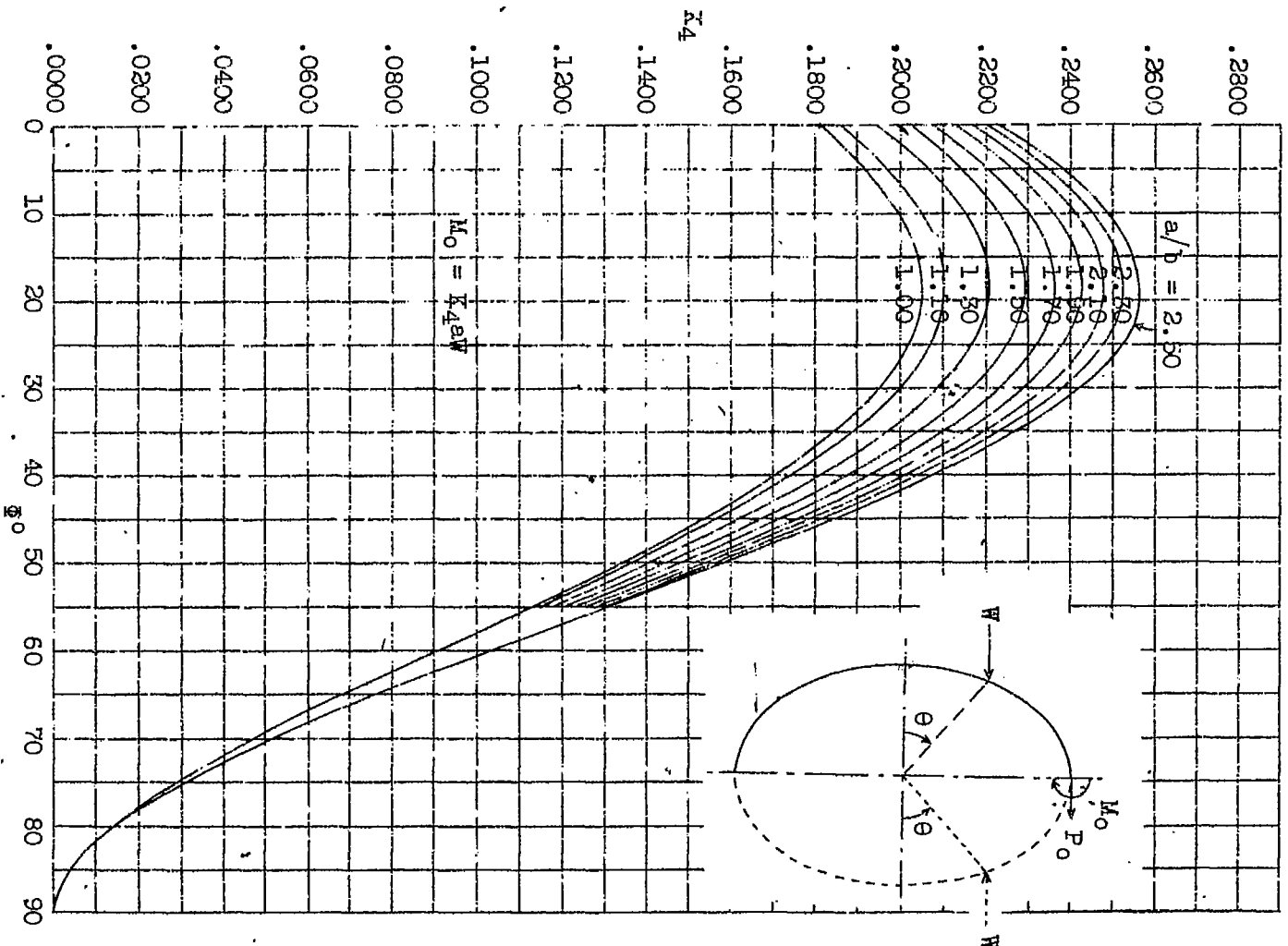
Figure 4.— Thrust coefficients for case B.

Figure 5.-- Moment coefficients for case B.
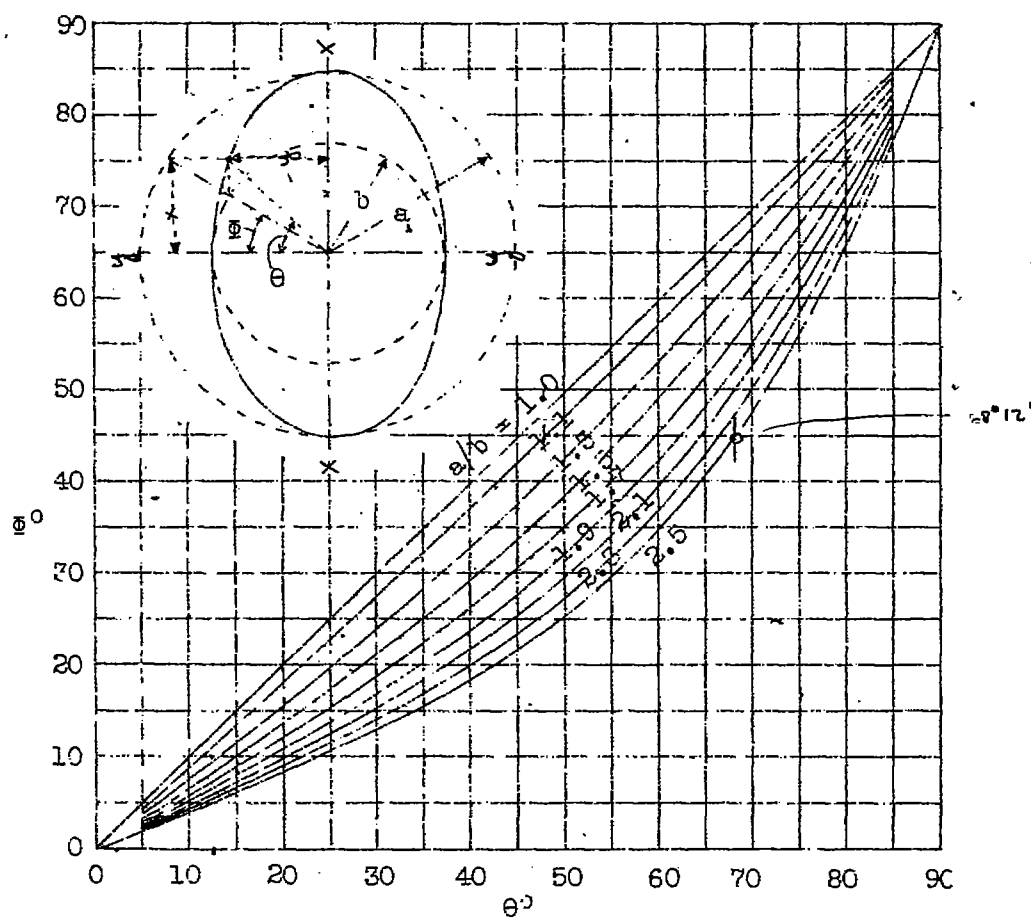
Fig. 5

$x = a \sin \phi$

$y = b \cos \phi$

Figure 6.- Values of Φ.